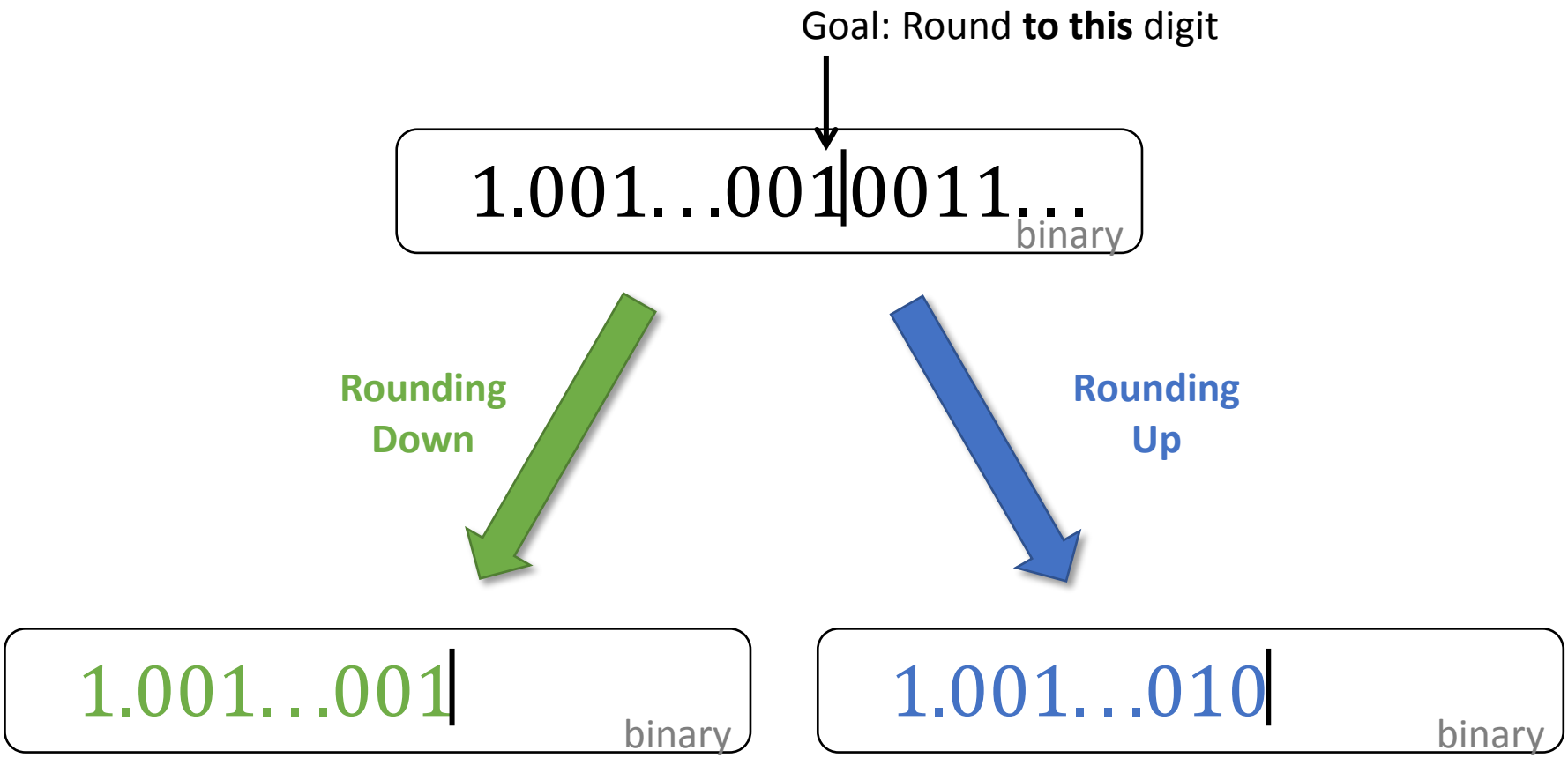
Binary Rounding

Rounding

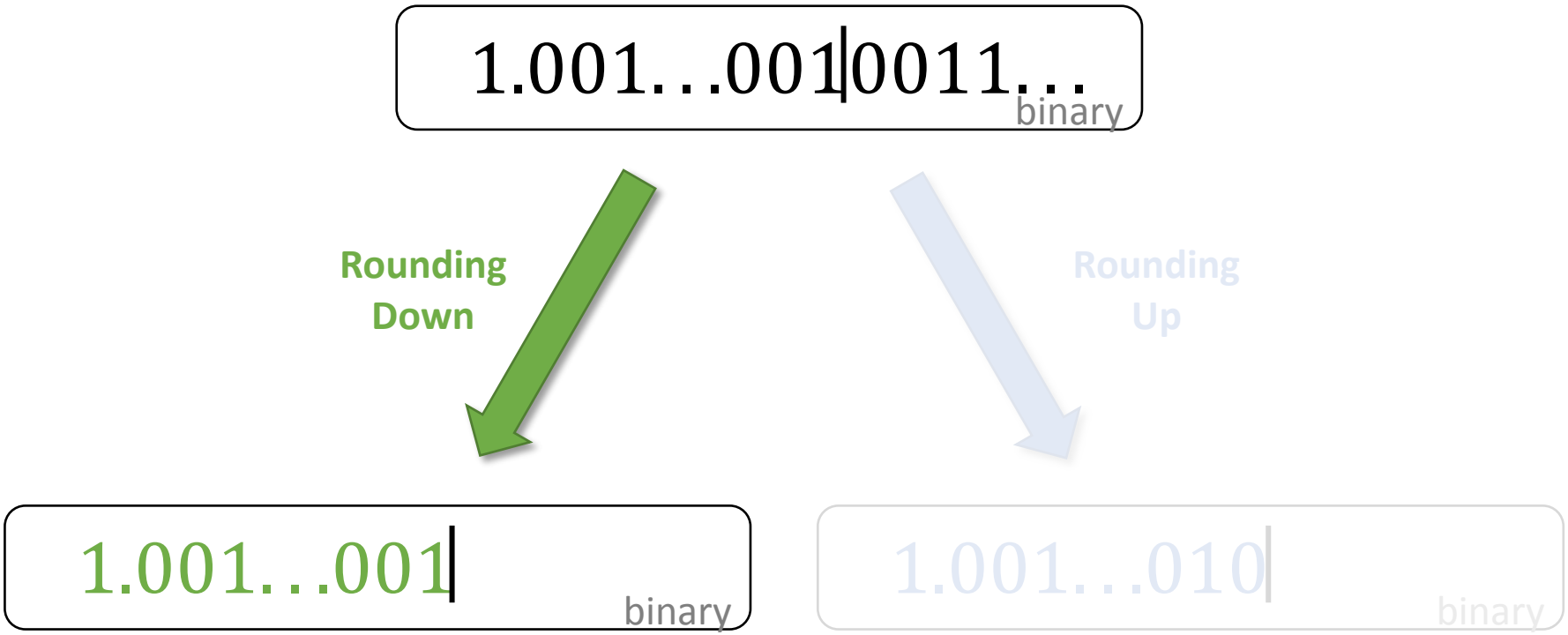
- Situation:
  Number lies **between** two representable values

- Question:
  What rounding choices do we have?

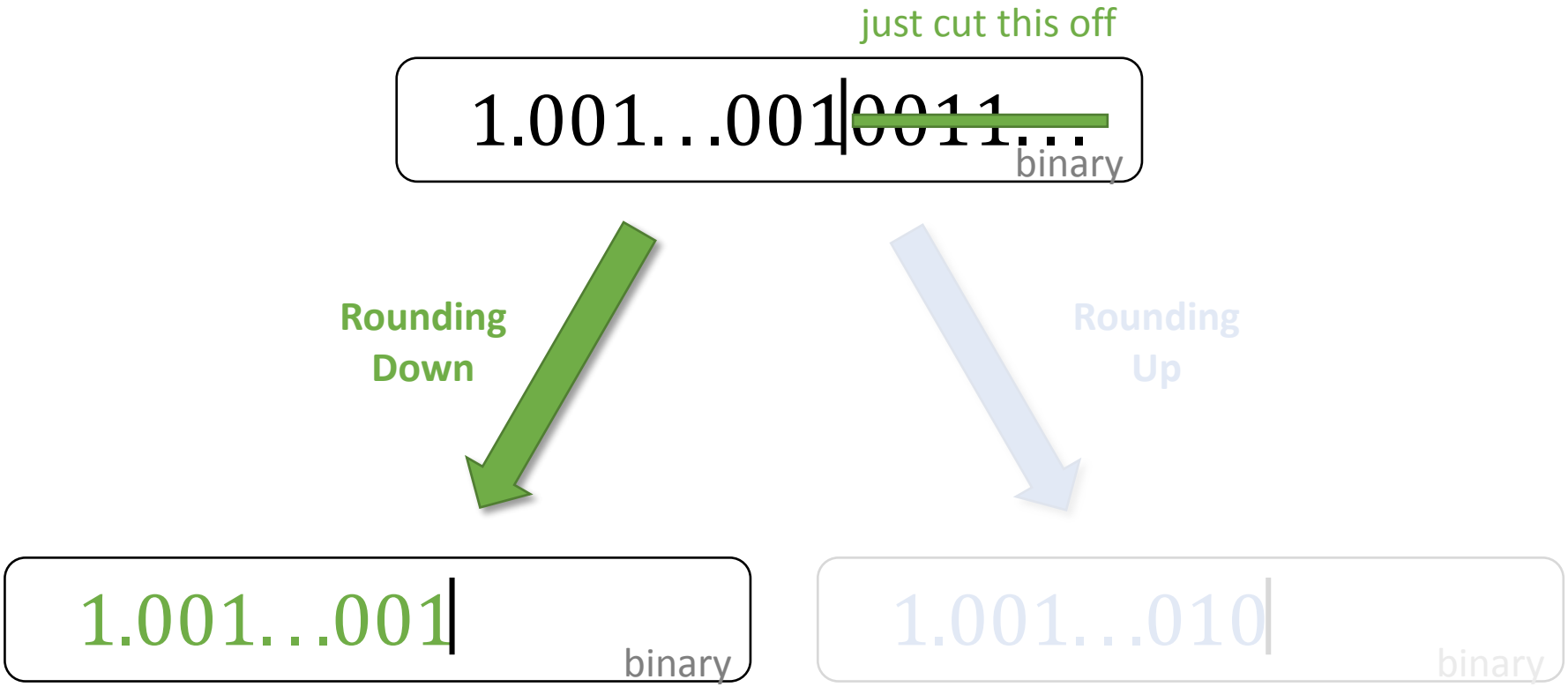
Rounding – Choices



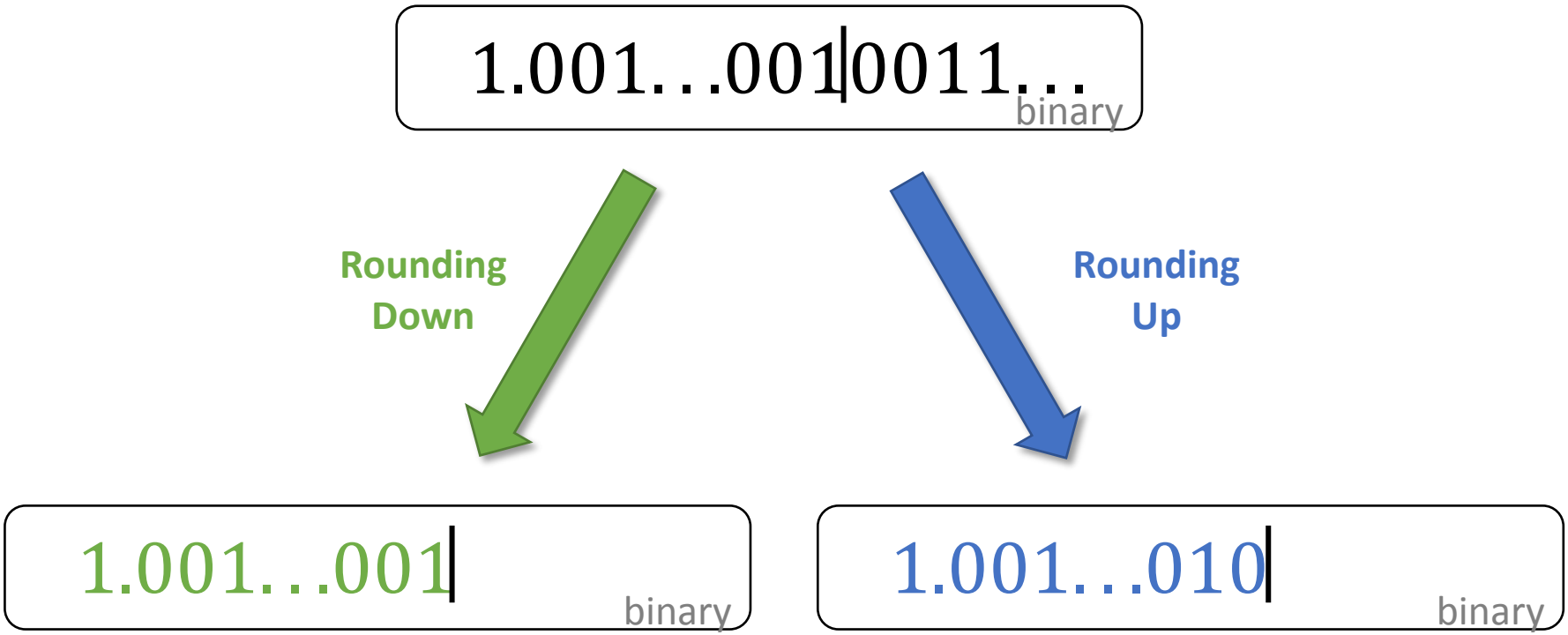
Rounding – Choices



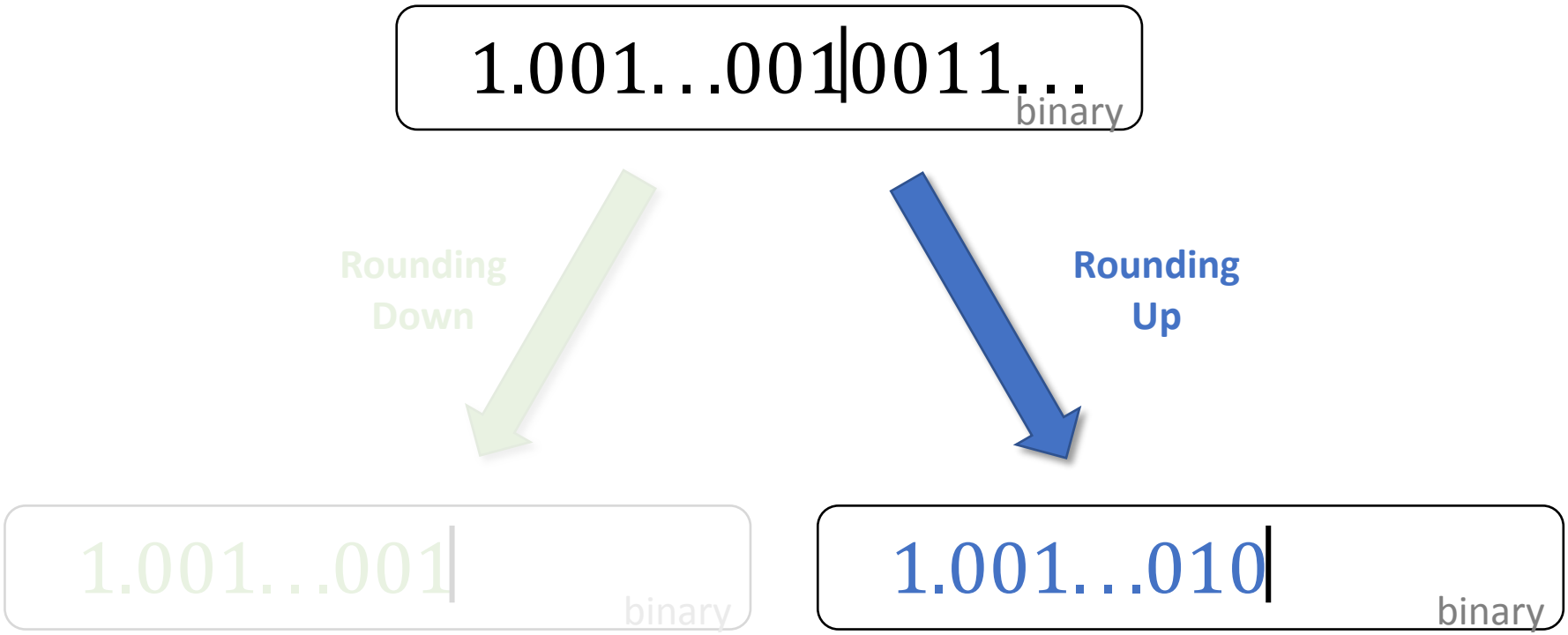
Rounding – Choices



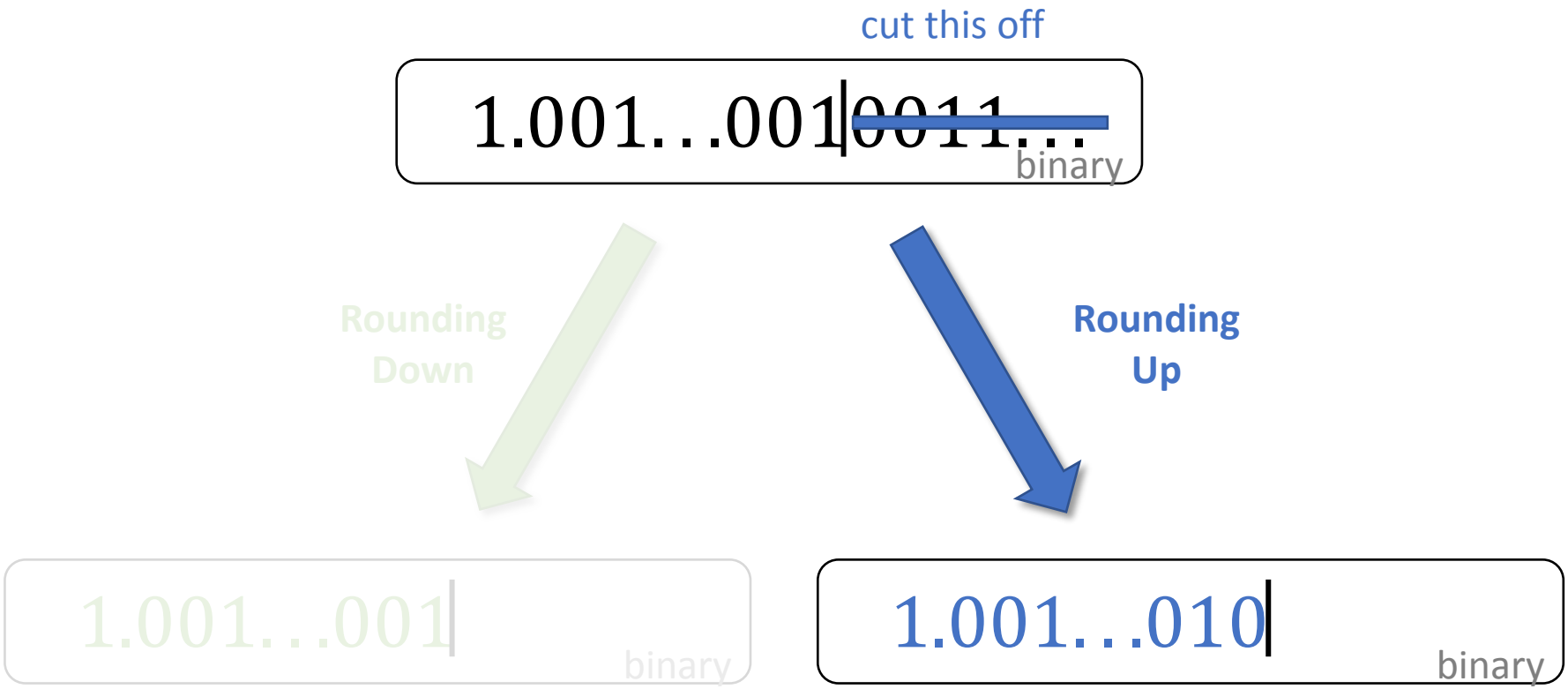
Rounding – Choices



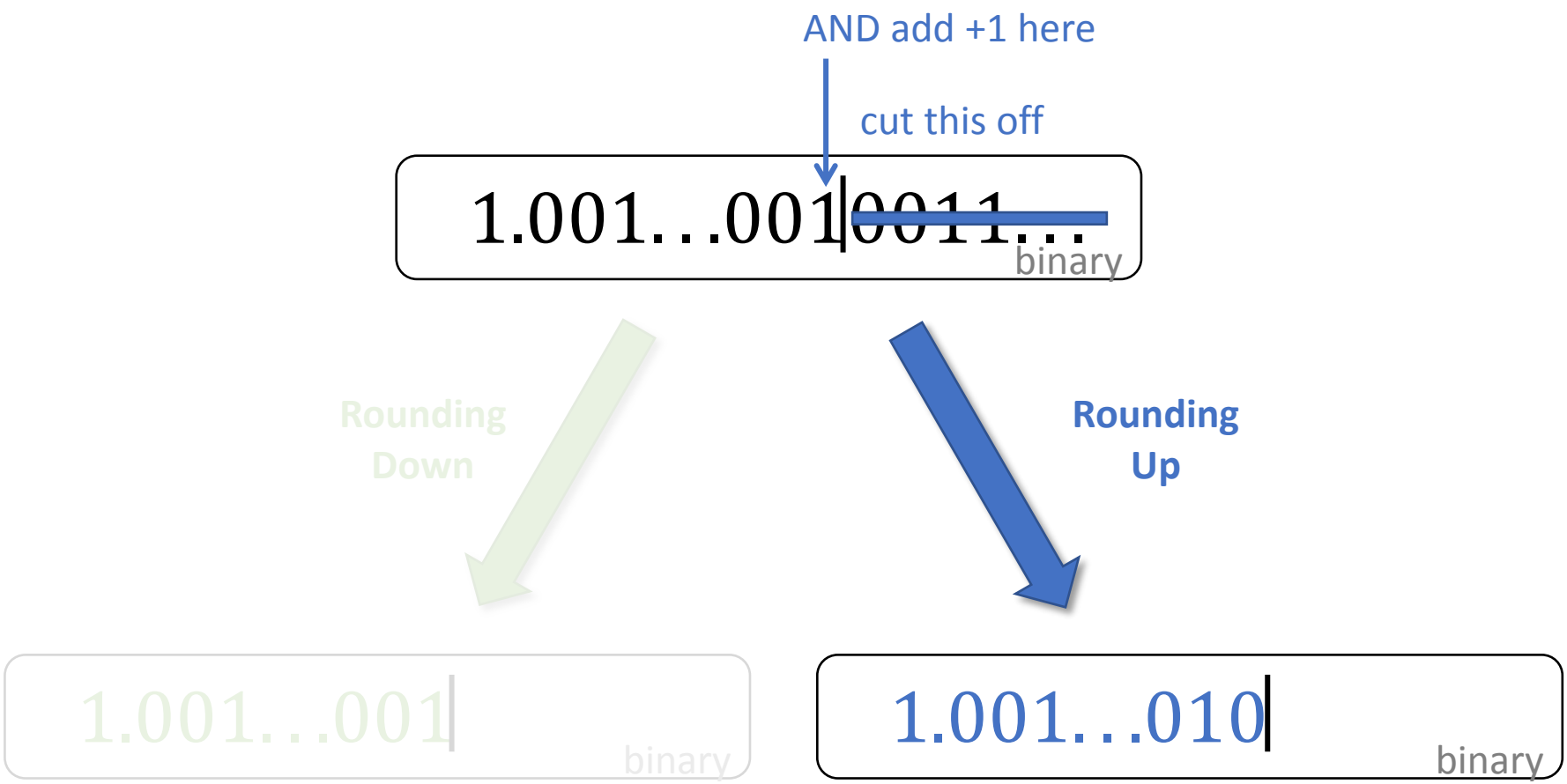
Rounding – Choices



Rounding – Choices



Rounding – Choices



Which when?

- Question:

  When does the computer round up/down?

- Rule:

  Computer rounds to the **closest representable number**.

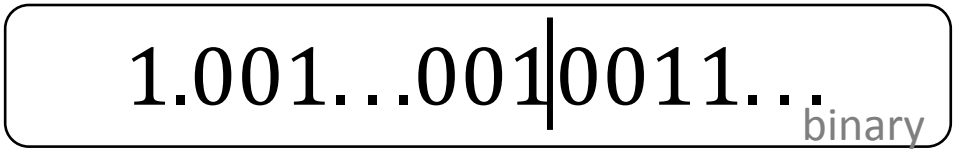
Which when?

- **Case 1:**

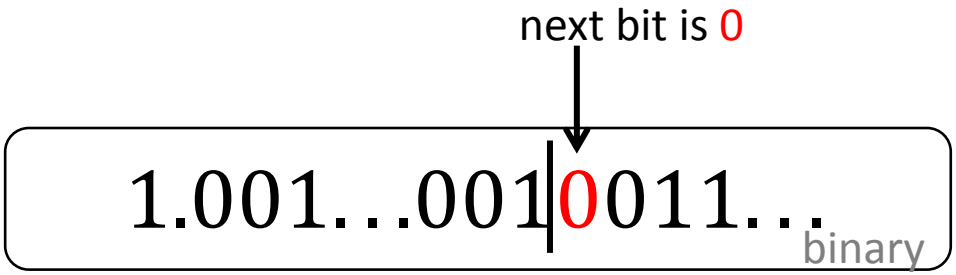
  next bit is **0**

  → round down

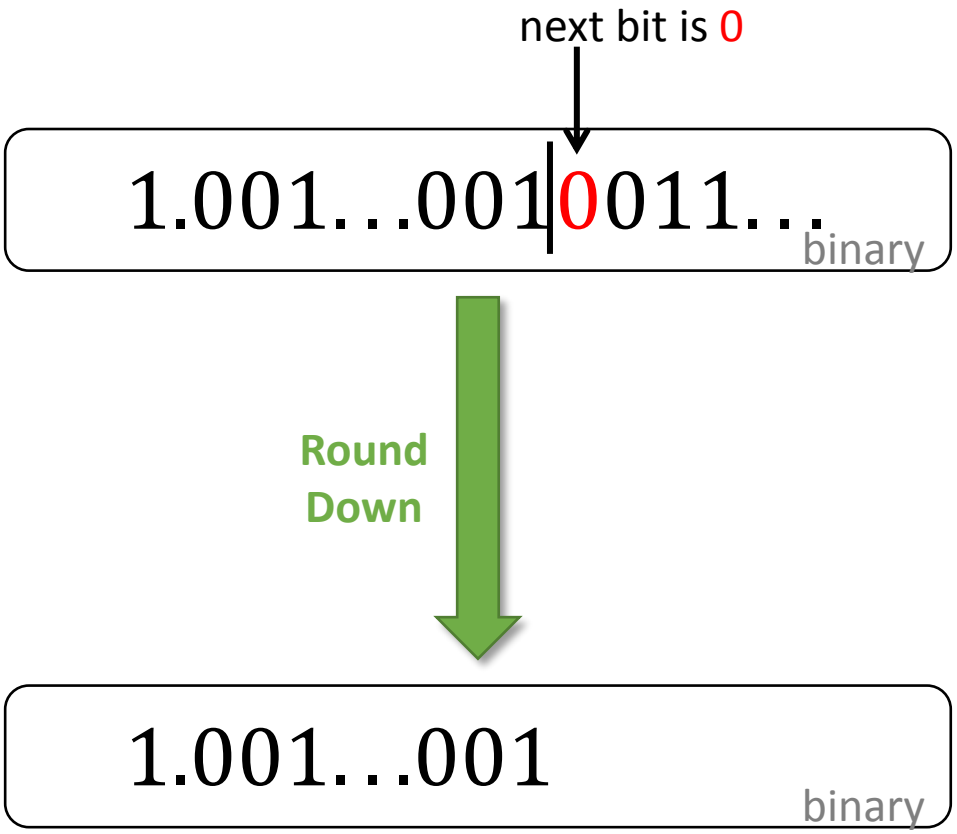
Case 1 – Example



Case 1 – Example



Case 1 – Example



Which when?

- **Case 1:**

  next bit is **0**

  → round down

- **Case 2:**

  next bit is **1** AND

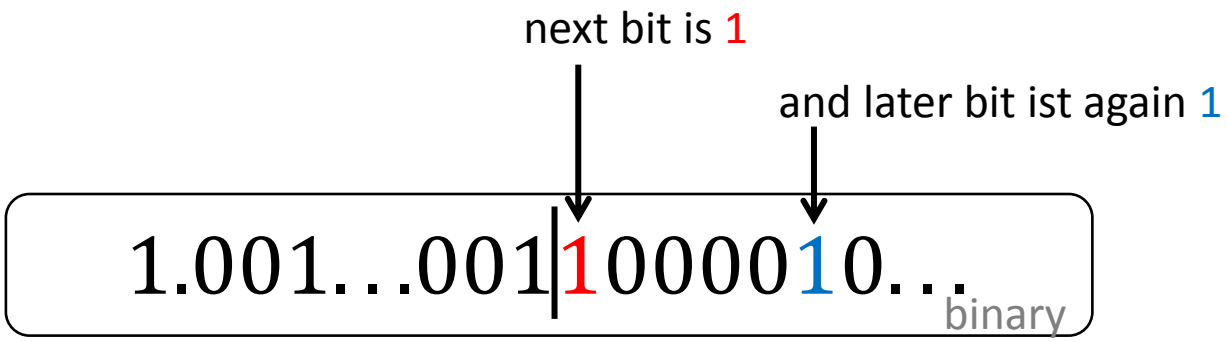
  at least one **later** bit is **1** again

  → round up

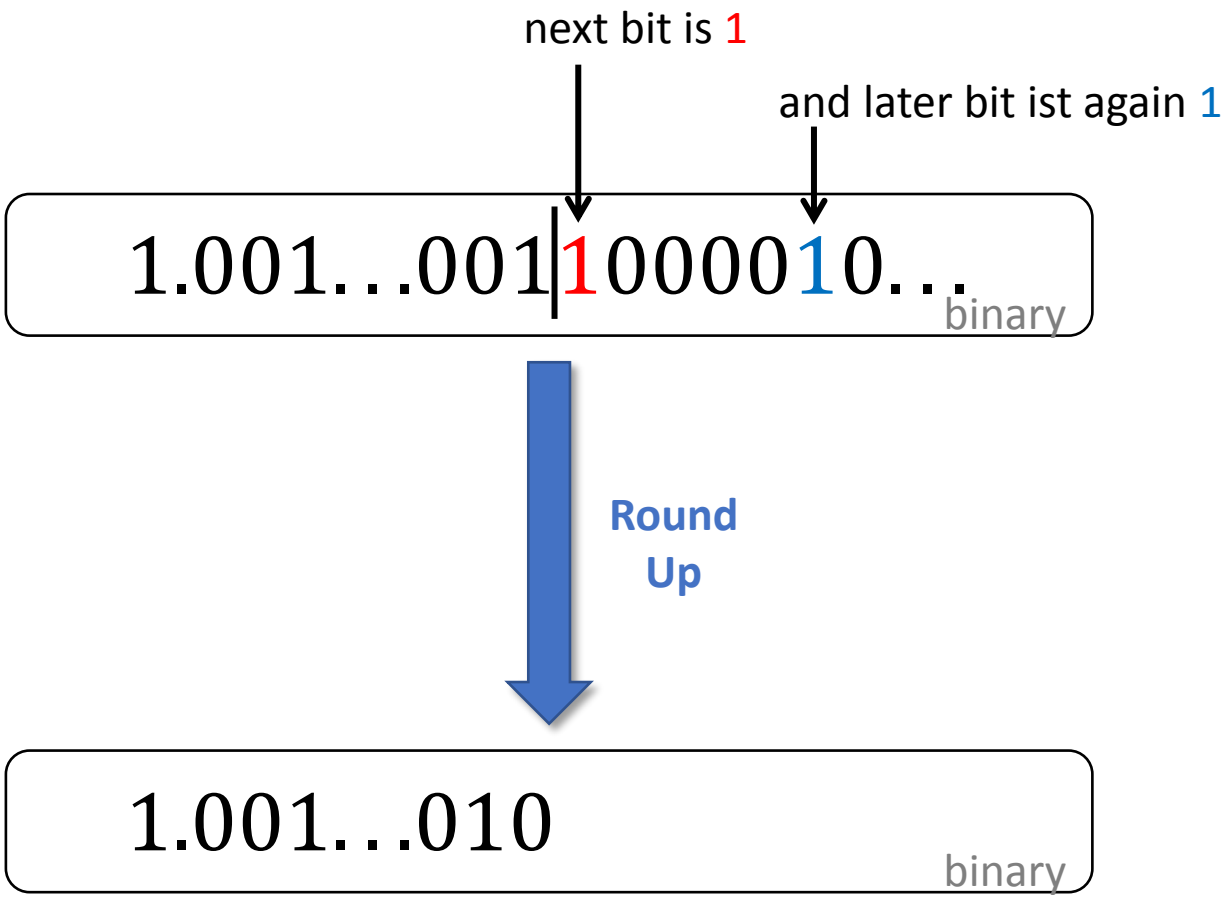
Case 2 – Example

$1.001\ldots001|1000010\ldots$ binary

Case 2 – Example



Case 2 – Example



Which when?

- **Case 1:**
  next bit is **0**
  → round down

- **Case 2:**
  next bit is **1** AND
  at least one **later** bit is **1** again
  → round up

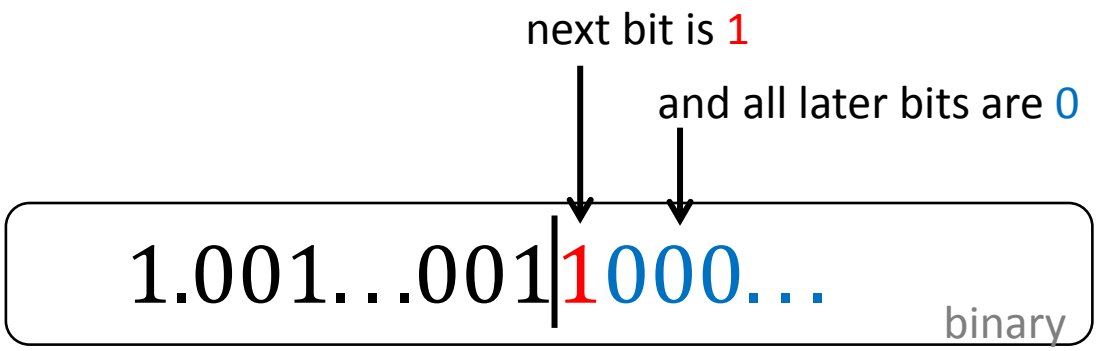
- **Case 3:**
  next bit is **1** AND
  all the **following** bits are **0**
  → round so that last bit is 0 (*)

(*) Round-to-Even Rule

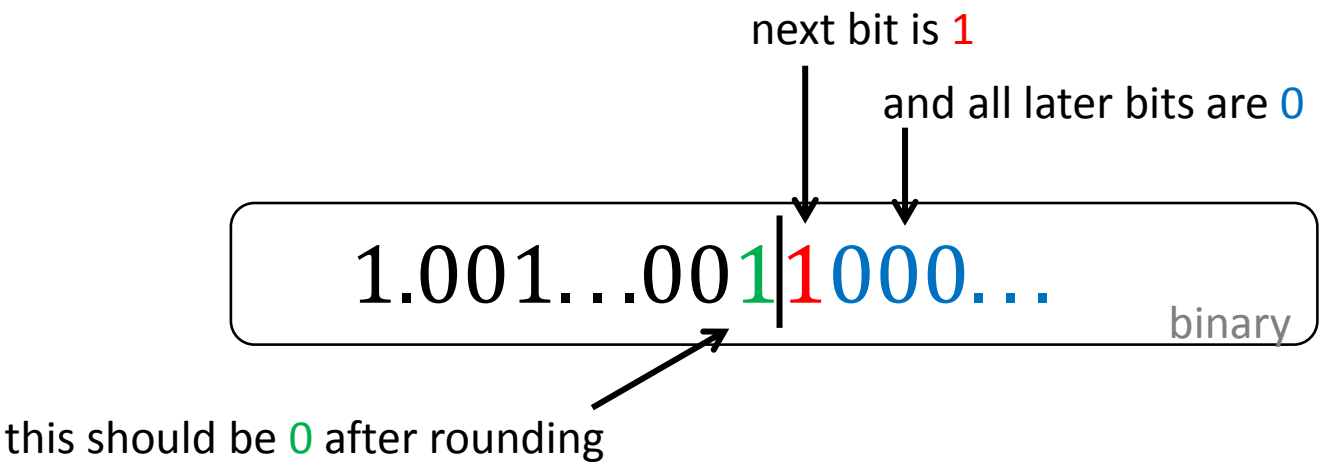
Case 3 – Example 1

$1.001\ldots001|1000\ldots$ binary

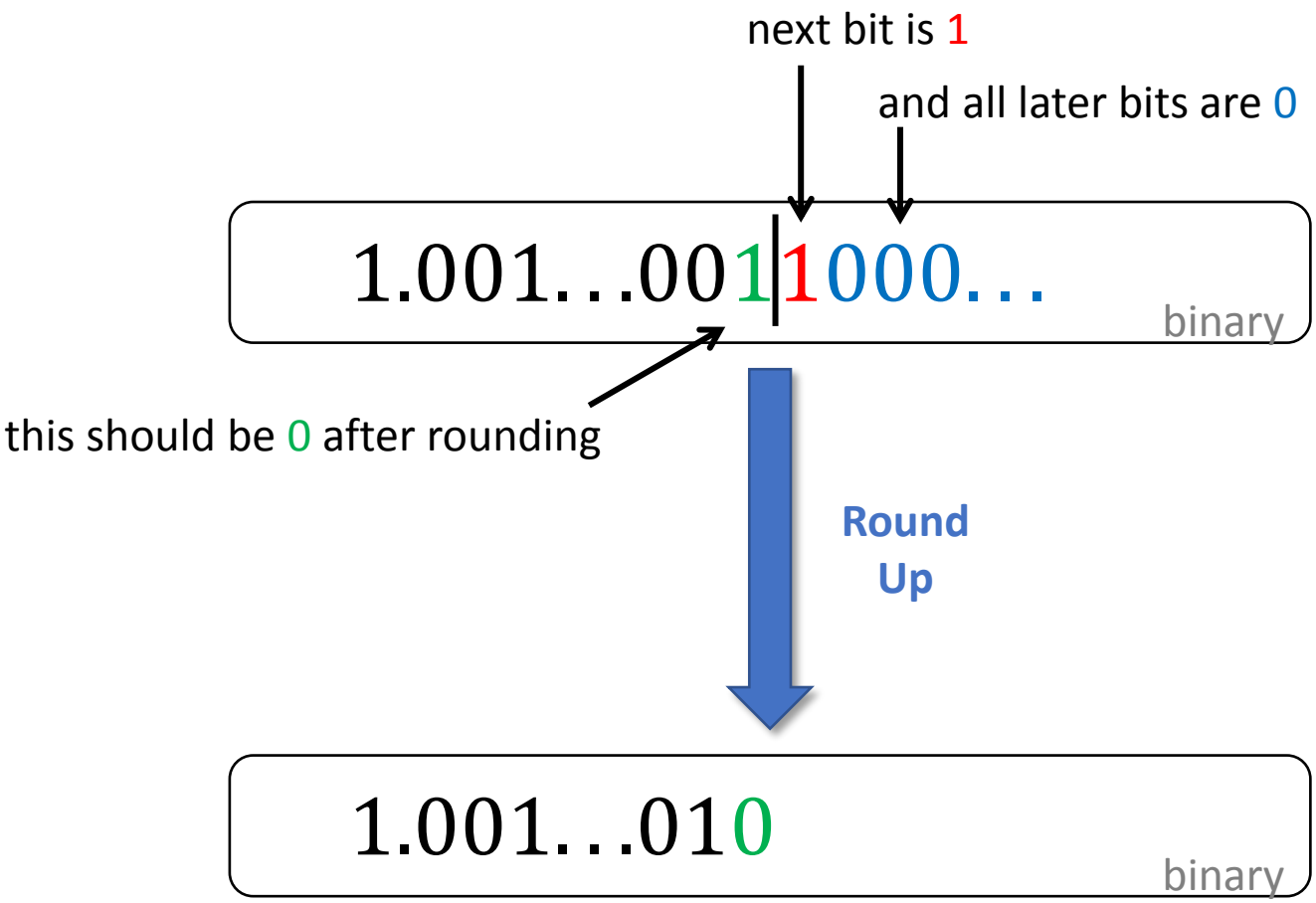
Case 3 – Example 1



Case 3 – Example 1



Case 3 – Example 1

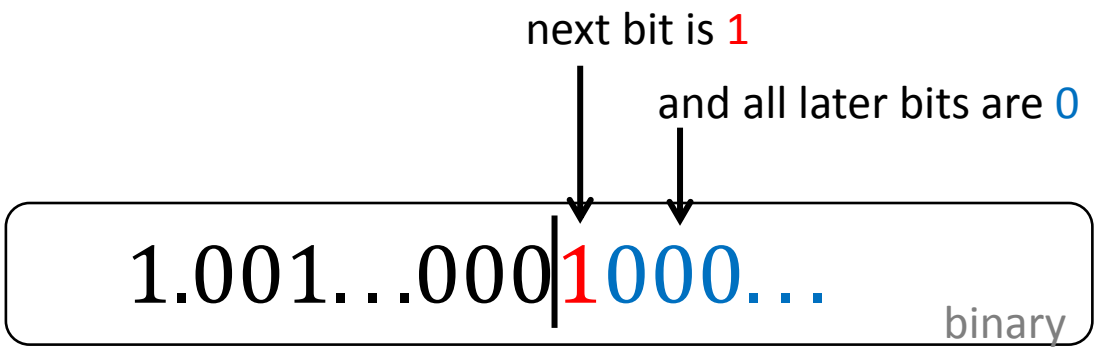


Case 3 – Example 2

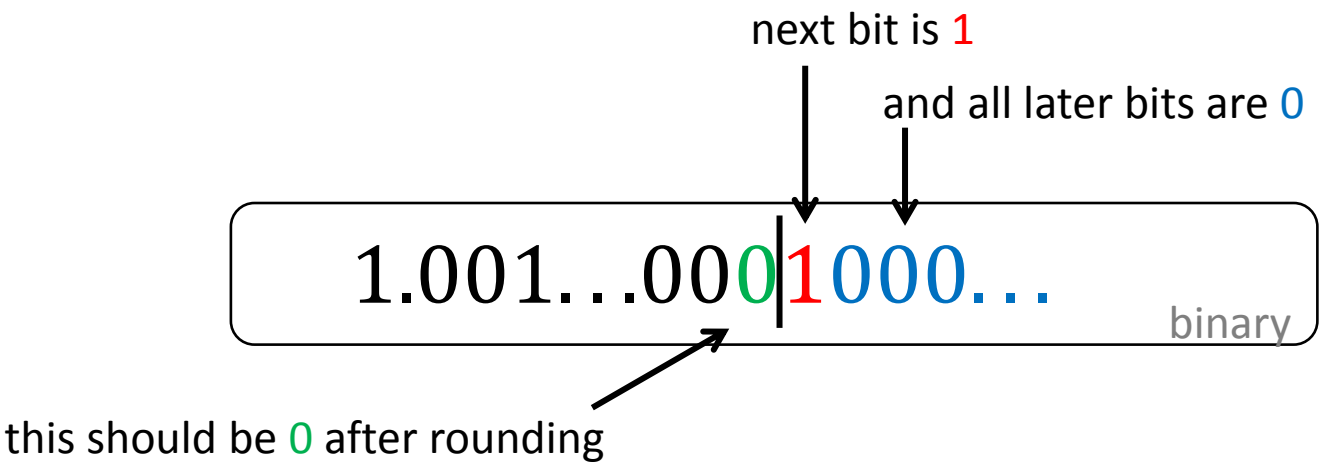
1.001…000|1000…

binary

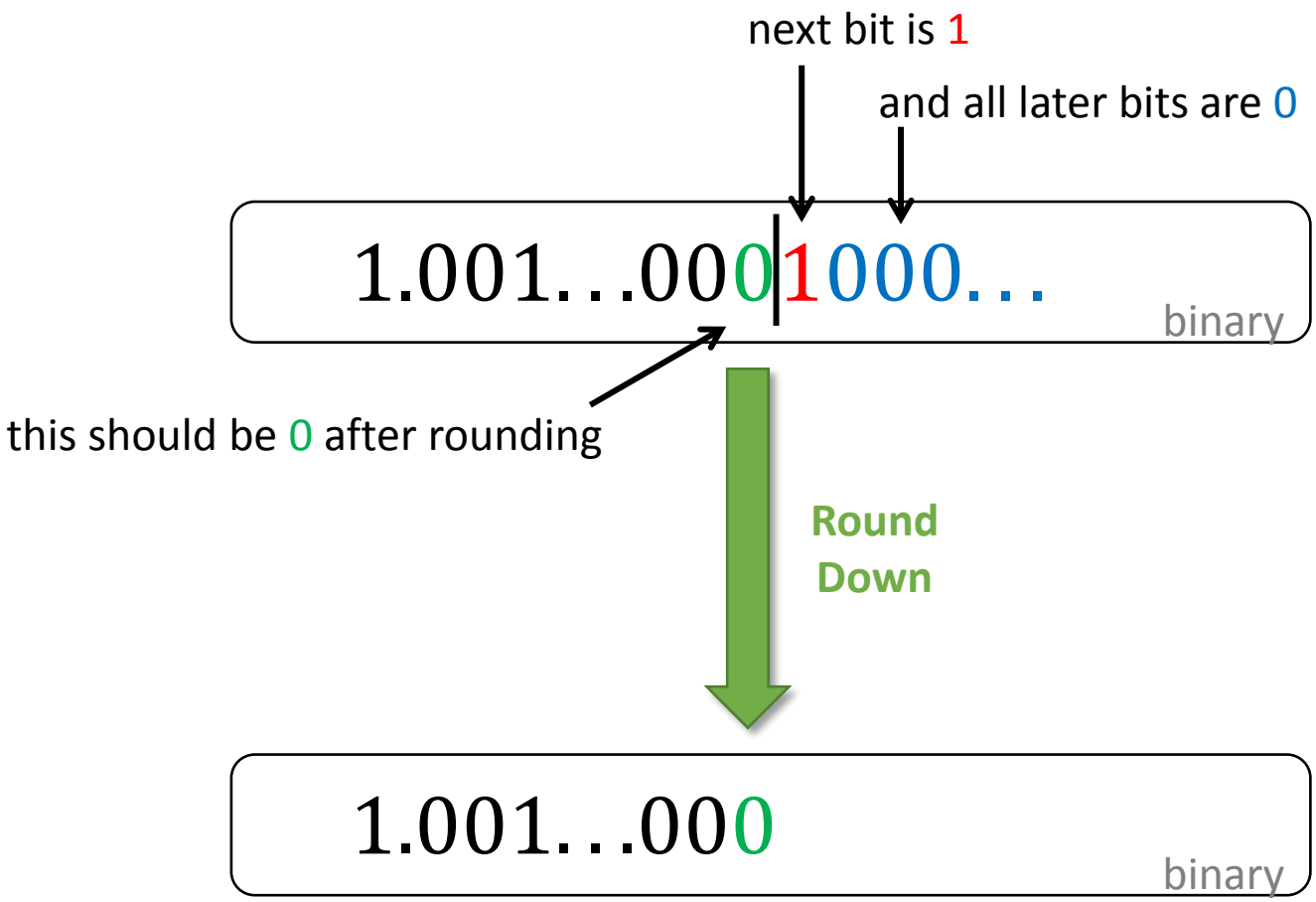
Case 3 – Example 2



Case 3 – Example 2



Case 3 – Example 2



Why Round-to-Even?

- Reason:

  Round down in approx. 50% of all times and round up in the remaining 50%.

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  Always round down in Case 3

  → **underestimation**

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  → **underestimation**

  Always round up in Case 3

  → **overestimation**